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INDIVIDUAL INCOME, INCOMPLETE INFORMATION, AND AGGREGATE CONSUMPTION

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Abstract

Individual income is much more variable than aggregate per capita income. I argue that aggregate information is therefore not very important for individual consumption decisions and study models of life-cycle consumption in which individuals react optimally to their own income process but have incomplete or no information on economy wide variables. Since individual income is less persistent than aggregate income consumers will react too little to aggregate income variation. Aggregate consumption will be excessively smooth. Since aggregate information is slowly incorporated into consumption, aggregate consumption will be autocorrelated and correlated with lagged income. On the other hand, the model has the same prediction for micro data as the standard permanent income model. The second part of the paper provides empirical evidence on individual and aggregate income processes and calibrates the model using the estimated parameters. The model predictions qualitatively correspond to the empirical findings for aggregate consumption but do not match them well in magnitude.

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1. Introduction

Contrary to the predictions of the modern version of the permanent income hypothesis (Hall, 1978), aggregate consumption changes in the U.S. are correlated with lagged income changes (see Flavin, 1981). Moreover, Deaton (1987) and Campbell and Deaton (1989) demonstrated that consumption is smoother than predicted by the model if income follows a highly persistent process. In individual data, on the other hand, the orthogonality condition implied by the permanent income model is much harder to reject as a multitude of recent studies shows.[2] If it is true that the model holds for individual data but not for aggregate data[3] then some type of aggregation bias should explain the differences.

A variety of possible biases have been explored. Finite lifetimes will introduce a dependence of consumption on cohort characteristics at the aggregate level and the martingale result found by Hall will not hold. Galí (1990) has developed this point in a recent paper and shown that it is not important enough empirically to explain aggregate consumption data. Attanasio and Weber (1993) have stressed nonlinearities as a possible reason for excess sensitivity at the aggregate level. Finally, a recent paper by Goodfriend (1992) suggests that agents may lack contemporaneous information on aggregate variables which invalidates the martingale property of the model at the aggregate level. In this paper I explore the theoretical and empirical implications of this type of incomplete information.

It is not unlikely that aggregate information plays little role in household decisions since the economic environment in which individuals operate differs sharply from the economy as it is described by aggregate data. Most importantly, individual income is much more variable than aggregate income: Below, I estimate that the standard deviation of quarterly household level income changes is about thirty times larger than that for aggregate per household income. While some of this variation will be attributable to measurement problems, a large part should reflect idiosyncratic

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2 See Deaton (1992) for a recent survey of the literature.

3 The inability to reject the model in micro data may of course also stem from problems related to measurement error, inexact variable definitions, etc. that make these tests less powerful.

income shocks. Therefore, individuals may make little effort to gather information on the behavior of the economy, but rather watch only their own prospective fortunes. Furthermore, individual income processes are much less persistent than aggregate income. The optimal consumption response calculated on the basis of individual income processes differs substantially from the predictions of a representative agent model calibrated with aggregate data. Using these facts, I construct a simple model in which agents react optimally to their individual income innovations but do not incorporate information on economy wide variables. The model correctly predicts what we observe in aggregate data: the correlation of consumption changes with lagged income and excess smoothness.

A simple example makes clear how the model works. Suppose a worker gets laid off from his job; he does not know immediately whether this is due to specific conditions at his firm or because of the onset of a general recession. If the layoff is due to highly individual factors then it will be easy for the worker to find new employment and the income reduction associated with the unemployment spell does not call for a major revision in consumption expenditures. Should the unemployment be due to aggregate factors, employment will be depressed at other firms as well and lead to a much longer expected unemployment spell. The necessary revision in consumption will be much larger than in the former case. The worker adjusts consumption in a way that will be correct on average given his overall experience with unemployment.

Looking at aggregate data, an econometrician will find *ex post* that everybody revised consumption downward too little at the onset of a recession. Subsequently, there will be further revisions once workers learn about the true scope and persistence of the shock. Consumption will appear correlated with lagged income and will appear smoother than predicted by a model where agents know the cause and length of their unemployment spell immediately.

There are a number of well known applications of the idea that individual agents may have incomplete aggregate information. Phelps (1969) and Lucas (1973) suggested a model in which workers/suppliers confuse aggregate and relative price movements. This yields an observable Phillips curve relationship in aggregate data which is not predicted by a full information representative agent model. Altonji and Ashenfelter (1980) use the same feature in a life-cycle model

of labor supply to generate an intertemporal substitution effect. If the aggregate wage follows a random walk and agents have full information there is no room for intertemporal substitution. If workers only know the lagged aggregate wage and their own wage, consisting of an individual and an aggregate component, then the model yields aggregate employment fluctuations even if the aggregate wage is a random walk. Froot and Perold (1990) have recently suggested a model where securities market specialists observe only information on their own stock contemporaneously but not aggregate information. Their model yields correlated aggregate stock returns.

In all of these models agents observe the aggregate variable with a one period lag. An analogous model in which agents learn about aggregate income with a one quarter delay has been suggested for consumption behavior by Goodfriend (1992). His model yields an MA(1) process for consumption changes. Therefore, no variable lagged at least twice should be able to predict consumption changes. The hypothesis of lagged information on income has been considered informally by Holden and Peel (1985). They reject this model on U.K. data by regressing consumption changes on income and consumption lagged twice. Campbell and Mankiw (1989) use information variables lagged at least two periods and find the same result for the U.S. and other countries.

Models with lagged aggregate information are usually motivated by the fact that aggregate variables like GNP only become public with a lag of about a quarter. Appealing to a publication lag alone is unappealing in Goodfriend's model, however. Prices will aggregate the information of individuals perfectly in a rational expectations models were every individual costlessly receives a small piece of the total information (Grossman, 1981). In the standard permanent income-consumption model this information would be transmitted in the price of the one asset traded in the economy. Thus, agents would only need to observe their own income as well as the interest rate. On the other hand, good arguments could be made why the conditions for a fully revealing rational expectations equilibrium do not hold and asset prices at best serve as noisy signals for aggregate income.

I prefer a slightly different interpretation of the incomplete aggregate information models that I will present: agents may simply not care enough about aggregate information because ignoring it is not very costly for (most) households. Therefore in this paper I examine Goodfriend's model with lagged information on aggregate income as well as a version where agents know only their own

income processes but never observe the aggregate component in their income. The latter feature has also been used by Deaton (1991) in a model of precautionary savings and liquidity constraints. To avoid convoluting information aggregation with other issues, I use Flavin's (1981) model with quadratic instantaneous utility as a tool for this analysis. This allows explicit solutions for the consumption process. Given the joint behavior of income and consumption it is then possible to calculate the regression coefficient of consumption changes on lagged income changes and the ratio of the variability in consumption to the variability in the income innovation. These predictions are easily compared to the sample statistics for aggregate data.

To calibrate the model it is necessary to have information on aggregate and individual income processes. While some estimates for individual earnings are available in the literature they are not well suited for the present purpose. In particular, no estimates are available that utilize quarterly income information comparable to the sampling frequency of aggregate data. I use the 1984 Panel of the Survey of Income and Program Participation which contains monthly information on family income to construct the appropriate quarterly micro data. The estimates for the micro income process are adjusted for measurement error using two alternative models of response behavior.

Using these results, I find that the model yields predictions that are in the correct direction and deviate substantially from the full information case. Quantitatively, they do not match the results for U.S. aggregate data well, however. The model generally tends to predict too high a correlation of consumption with lagged income but not smooth enough consumption. Notice, however, that this procedure, using actual micro parameters to calibrate the model, subjects the model to a much more stringent test than is usually adopted in the macro consumption literature. I also show that rational consumers are unlikely to concern themselves with acquiring aggregate information as maintained by Goodfriend because the gain amounts to less than two Dollars every quarter.

The paper is organized as follows. In the next section, I review the basic full information model and the empirical failures it has generated. Using a simple income process as an example, section 3 analyzes the model with no observability of aggregate income and describes its implications. In section 4, I contrast this with the model of Goodfriend where aggregate information becomes available with a one period lag. The model implications of more general income processes are

discussed in section 5. The next two sections are devoted to the estimation of individual and aggregate income processes; section 7 also summarizes the stylized facts on the consumption puzzles.. Section 8 uses the estimates on the income processes to predict features of aggregate consumption and compares the results to the findings in the previous section. Section 9 concludes.

2. The Model with Complete Aggregate Information

In this section I will set up the model and review a simple example where agents have individual specific income processes that differ from the time series structure of aggregate income. However, each micro agent has full contemporaneous information on aggregate income. At the aggregate level, this model is equivalent to a representative agent model.

The consumer solves the life-cycle maximization problem:

$$\max_{\{c_t\}} E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\delta} \right)^{s-t} u(c_s) \tag{1}$$

$$\text{s.t. } W_{t+1} = (1+r)[W_t + y_t - c_t]$$

$$\lim_{t \to \infty} (1+r)^{-t} W_t = 0 \quad \text{a.s.}$$

c_t is consumption, y_t is non-interest income, and W_t is non-human wealth at the beginning of period t. Income is paid and consumption takes place before interest accrues on wealth. r and δ are the interest rate and the time discount rate, respectively. Both are assumed to be constant and consumers can borrow and lend freely at the rate r.

Flavin (1981) has shown that a quadratic instantaneous utility function and $r = \delta$ yields the following relation for the change in consumption

$$\Delta c_t = r \sum_{s=0}^{\infty} \frac{(E_t - E_{t-1}) y_{t+s}}{(1+r)^{s+1}} \tag{2}$$

i.e. consumption changes equal the present value of the news about future income.

If income follows a univariate time series process known to the consumer then (2) can be used to relate changes in consumption to the innovations in the income process directly. Let income be a process that is stationary in first differences so that it has a Wold representation $\Delta y_t = A(L)\varepsilon_t$. For this process the change in consumption is given by

$$\Delta c_t = A\left(\frac{1}{1+r}\right)\varepsilon_t \tag{3}$$

I will consider models where all individuals have identical income *processes* while each agent faces different realizations of this process. To fix ideas, consider a simple example where income consists of a random walk with innovations that are common to all individuals and a white noise component with shocks that are uncorrelated across individuals. In first differences this process takes the form

$$\Delta y_{it} = \varepsilon_t + u_{it} - u_{it-1} \tag{4}$$

Subscripts *i* denote individual variables while no subscripts refer to aggregate variables. ε_t is the aggregate income innovation, and u_{it} is the individual income shock. The innovations are assumed to be uncorrelated.

Every period agents observe their own income y_{it} as well as aggregate income y_t. Given that they also know the complete history of these variables they can infer the fundamental shocks ε_t and u_{it}. What is relevant to the consumer is how much each process contributes to permanent income. Given (3), the optimal rule is to adjust consumption fully to the permanent (aggregate) shock and by the annuity value $r/(1+r)$ to the transitory (individual) shock, i.e.

$$\Delta c_{it} = \varepsilon_t + \frac{r}{1+r} u_{it} \tag{5}$$

The change in average per capita consumption is found by summing over individuals. Because the individual shocks are mutually uncorrelated they will sum to zero in a large population so that we obtain

$$\Delta c_t = \frac{1}{n}\sum \Delta c_{it} = \varepsilon_t \tag{6}$$

Aggregate consumption is a random walk and the consumption change is just the aggregate income innovation. Hence this model yields the same predictions as a representative agent model where the representative agent faces the aggregate income process $\Delta y_t = \varepsilon_t$. In particular, consumption changes are uncorrelated with lagged aggregate variables, like lagged consumption or income changes. This martingale property has been tested by Hall (1978) by regressing consumption changes on lags of consumption, income, and stock prices. Hall found little explanatory power for income but rejected nonpredictability for stock prices. I will call this rejection of the full information model the *orthogonality failure.*

Hall's test only exploits the information contained in the Euler equation. Combined with the budget constraint the model has the additional implication that the variance of consumption changes should depend on the structure of the income process as pointed out by Deaton (1987). Taking variances in (3) and applying the formula to the representative agent model with random walk income yields

$$\frac{\sigma_{\Delta c}}{\sigma_\varepsilon} = A\left(\frac{1}{1+r}\right) = 1 \tag{7}$$

since $A(z) = 1$ for the random walk. The ratio of the standard deviation of consumption changes to the standard deviation of income innovations should equal the consumption response predicted by the model, one in this case. Deaton found that the empirical equivalent of this variance ratio is actually much too low based on an AR(1) for the first differences in aggregate income. Thus consumption exhibits *excess smoothness.*[4]

4 Campbell (1987) provides a joint test of the orthogonality and smoothness conditions of the model. For the full information example it implies that a VAR of aggregate income changes and aggregate savings has the following form:

$$\begin{bmatrix} \Delta y_t \\ s_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & (1+r) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix}$$

In the general case, the restrictions on the VAR are equality of the coefficients in the first column of the coefficient matrix and the difference of the coefficients in the second column being $(1+r)$.

Notice how Quah (1990) has used a representative agent model with an income process as in (4) to generate excess smoothness. Agents behave just as in (5) but both shocks ε_t and u_t are common across individuals. The econometrician only observes the compound income process and calculates the magnitude of the optimal consumption change based on this (misspecified) model. Quah demonstrates that the econometrician's model implies a more variable consumption series than the true series and therefore apparent excess smoothness. However, since consumption in (5) is uncorrelated with any lags of income this cannot account for the orthogonality failure also present in the data.[5]

Using the simple example above, I will now address how incomplete information of agents on aggregate income can lead to both the orthogonality failure and excess smoothness at the aggregate level. A more general treatment will follow.

3. Unobservable Aggregate Shocks

Consider the income process in (4) again but now assume that individuals can only observe y_{it}. If the individual cannot distinguish the aggregate and the individual component then this process to her looks just like an MA(1) process for the first differences in income. The income process the individual observes can thus be written as

$$\Delta y_{it} = \eta_{it} - \theta \eta_{it-1} \tag{8}$$

The random variable η_{it} will contain information on current and lagged aggregate and individual income innovations. Note that $\{\eta_{it}\}$, though not a fundamental driving process of the model, is an innovation sequence with respect to the history of individual income changes. Muth (1960) has

5 Campbell's (1987) test is robust to the type of superior information by consumers as envisioned by Quah. However, this test clearly rejects the model with U.S. aggregate data.

shown that $(1-\theta)\eta_{it}$ is the optimal predictor of the innovation to the random walk component of income. The MA parameter θ in (8) depends on the relative variances of the aggregate and individual income shocks.[6]

Equation (3) still holds so that changes in individual consumption follow

$$\Delta c_{it} = \left(1-\frac{\theta}{1+r}\right)\eta_{it} = \frac{1+r-\theta}{1+r}\eta_{it} \equiv A\eta_{it} = A\frac{\Delta y_{it}}{1-\theta L}. \tag{9}$$

Individual consumption changes are a martingale with respect to the history of individual consumption and income. A researcher doing Hall's (1978) analysis on panel data for individuals should not reject the permanent income model.[7] This type of testing procedure has been carried out, for example, by Altonji and Siow (1987) who do not reject the model. Estimating a structural model as in Hall and Mishkin (1982) would not be correct because their model assumes that consumers know the income components in (4).[8] The correct structural model would use the income process in (8) instead. This has been pointed out by Speight (no date) who finds support for the model with incomplete information on Austrian panel data while the Hall and Mishkin model is rejected.

I want to focus here on the aggregate implications of the incomplete information case. To find the change in average per capita consumption use the last equality in (9) and equation (4) and sum over individuals.

$$\frac{1}{n}\sum\Delta c_{it} = \frac{A}{n}\sum\frac{\Delta y_{it}}{1-\theta L} = \frac{A}{n}\sum\frac{\varepsilon_t + u_{it} - u_{it-1}}{1-\theta L} \tag{10}$$

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6 Define the first order autocorrelation coefficient in (4) $\rho = -\sigma_u^2/(\sigma_\varepsilon^2 + 2\sigma_u^2)$. Then $\theta = -(1-\sqrt{1-4\rho^2})/2\rho$.

7 The martingale property only holds with respect to variables that are in individuals' information sets. Many researchers using panel data control for macroeconomic shocks. Goodfriend (1992) pointed out that such controls also invalidate the Hall procedure. I show below that the variance of individual income innovations is far larger than the variance of the aggregate component; this will therefore not be very important in practice.

8 This is not literally true. Hall and Mishkin (1982) only distinguish a permanent and a transitory income component. These are not identified with aggregate and individual income processes as in the example in the text. Furthermore, Hall and Mishkin find nonzero correlations between consumption changes and lagged income changes or lagged consumption changes in their data. Apart from the appropriateness of the structural income process it is these correlations that lead to a rejection of the model in their sample.

Individual shocks will sum to zero again so that we obtain

$$\frac{1}{n}\sum \Delta c_{it} = \Delta c_t = A\frac{\varepsilon_t}{1-\theta L}$$

$$\Delta c_t(1-\theta L) = A\varepsilon_t \tag{11}$$

Equation (11) has a number of interesting implications. Unlike individual consumption, the per capita series of consumption is not a random walk as the representative agent model predicts. Consumption now follows an AR(1) in first differences. The intuition for this is rather simple. Suppose an aggregate shock hits the economy. All the individual consumers see their income changing but they assume that a part of the shock is idiosyncratic and therefore transitory. They will change their consumption but not by as much as the permanence of the shock calls for. Because the shock is persistent, in the following period they will be surprised again that their income is higher than expected, they will increase their consumption further and so on.

All this implies that an econometrician working with the representative agent model will find both the orthogonality failure and the smoothness result in aggregate data. Suppose the econometrician estimates the following model

$$\Delta c_t = \alpha + \beta \Delta y_{t-1} + e_t \tag{12}$$

If the data are generated by (11) the expected value of β would be

$$\beta = \frac{cov(\Delta c_t, \Delta y_{t-1})}{var(\Delta y_{t-1})}$$

$$= \frac{E\left\{A\left(\frac{\varepsilon_t}{1-\theta L}\right)\varepsilon_{t-1}\right\}}{\sigma_\varepsilon^2} = \frac{A\theta\sigma_\varepsilon^2}{\sigma_\varepsilon^2} = A\theta \tag{13}$$

Because individuals do not recognize an aggregate shock to be permanent they will not adjust their consumption by as much as they would if it were the only type of shock to occur. This will lead

to more smoothness in aggregate data than predicted by the full information model where the variance of consumption changes equals the variance of aggregate income innovations. For the model with heterogeneous agents and incomplete information we get instead from (11)

$$\frac{\sigma_{\Delta c}}{\sigma_\varepsilon} = \frac{A}{\sqrt{1-\theta^2}} \tag{14}$$

If idiosyncratic shocks are present and the interest rate is small enough the ratio of the standard deviations of the change in consumption and the aggregate income innovation will always be less than one. To see this more clearly, consider the case where $r \to 0$. In this case $A = 1-\theta$ and (14) can be expressed as

$$\lim_{r \to 0} \frac{\sigma_{\Delta c}}{\sigma_\varepsilon} = \sqrt{\frac{1-\theta}{1+\theta}} \tag{15}$$

This will be less than one if $\theta > 0$.[9]

It is easy to see which features of the example drive the result. The representative agent model would hold for aggregate data if the aggregate and the individual income processes had the same persistence properties so that consumers would want to react in the same way to each type of shock. In this example, consumers do not want to increase consumption enough in response to an aggregate shock because they confuse it with the individual income innovation which is less persistent.

The results also hinge on the assumption that individuals cannot or do not find it profitable to distinguish aggregate and idiosyncratic shocks. Otherwise they would react differently according to the persistence properties of the specific shock observed. Goodfriend (1992) originally proposed

9 The simple example of the no information model implies that Campbell's (1987) VAR representation of income changes and savings has the form

$$\begin{bmatrix} \Delta y_t \\ s_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ C(L) & (1+r) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ (1-A)\varepsilon_t \end{bmatrix}$$

where the lag polynomial $C(L) = -\theta A(1-\theta L)^{-1}$. Thus it is the restriction of equality of the first column of the coefficient matrix rather than the second which is violated. This qualitatively reflects Campbell's findings.

such a model, where information on aggregate income becomes available with a one period lag. For comparison, I will present the implications of this model with lagged information on aggregate income in the following section.

4. Lagged Information about Aggregate Shocks

Suppose aggregate data are published with a one period lag. In period t individual i will observe y_{it} and the aggregate shock ε_{t-1}. Also assume again that the consumer has access to the infinite history of shocks and can therefore infer u_{it-1} as well once the aggregate shock is known. Write the income process (4) for the individual as

$$\Delta y_{it} = v_{it} - u_{it-1} \quad \text{where } v_{it} = \varepsilon_t + u_{it} \tag{16}$$

We can decompose the information the consumer gets every period into two parts. The first part is v_{it}, the current period innovation which is contained in current individual income y_{it}. The consumer does not know how the innovation in a particular period is composed of the permanent (aggregate) component and the transitory (individual) component. She will therefore attribute part of the current period innovation to each component given the relative variances. For every particular innovation there will be errors, of course. Secondly, the consumer gets information from the lagged aggregate shock. Once this information arrives she will be able to correct the error made last period in attributing the innovation to its components.

The optimal consumption response will have two parts corresponding to the two pieces of information: a response to the new innovation and a term that corrects for the error made in the previous period. The first part of the consumption response, the reaction to the current period innovation can be written as

$$\omega v_{it} + (1-\omega)\frac{r}{1+r}v_{it} = \frac{\omega+r}{1+r}v_{it} \tag{17}$$

where $\omega = \sigma_\varepsilon^2/(\sigma_\varepsilon^2 + \sigma_u^2)$ is the relative variance of the aggregate shock.[10] The first term is the proportion of the new innovation expected to be permanent, the consumption response to that part is one. The second term is the part expected to be transitory, the response is $r/(1+r)$.

Consider the correction for errors made last period. Define the negative of the error in the aggregate component as

$$\xi_{it-1} = \varepsilon_{t-1} - \omega v_{it-1} = \varepsilon_{t-1} - \omega(\varepsilon_{t-1} + u_{it-1}) = (1-\omega)\varepsilon_{t-1} - \omega u_{it-1} \tag{18}$$

The errors in the individual component and in the aggregate component have to sum to zero since the signal extraction problem the individual solved in *t-1* yielded unbiased predictors of the two components. The response of consumption in period *t* to errors made in *t-1* is therefore

$$(1+r)\left[\xi_{it-1} + \frac{r}{1+r}(-\xi_{it-1})\right] = \xi_{it-1} \tag{19}$$

The first term in the square bracket is the correction of the error in the aggregate component, the second term the correction for the error in the individual component. Notice that interest accrued on the portions of the shocks that had not been consumed in the last period.

Putting together the two parts of the total consumption response from (17) and (19) we obtain

$$\Delta c_{it} = \frac{\omega + r}{1+r} v_{it} + (1-\omega)\varepsilon_{t-1} - \omega u_{it-1} \tag{20}$$

Like in the model of the previous section, individual consumption changes still follow a martingale with respect to the history of individual income and consumption.[11] This can easily be seen by calculating the autocovariance $cov(\Delta c_{it}, \Delta c_{it-1})$. It will be proportional to $(1-\omega)\sigma_\varepsilon^2 - \omega\sigma_u^2$ which is zero. The lagged income innovations in (20) arise from the fact that errors are corrected after one period. However, optimal choice of the weight ω implies that these errors contain no information correlated with lagged income or consumption changes.

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10 Note that $\omega = (1+2\rho)/(1+\rho) = (1-\theta)^2/(1-\theta+\theta^2)$. It is much more convenient to work with ω here.

11 I thank Steve Zeldes for pointing out an error in a previous draft.

Sum the individual consumption responses in (20) for a large population to get the per capita consumption response

$$\Delta c_t = \frac{1}{n}\sum \Delta c_{it} = \frac{\omega + r}{1+r}\varepsilon_t + (1-\omega)\varepsilon_{t-1} \tag{21}$$

The change in aggregate consumption follows an MA(1) process. Notice that the impact response to an aggregate shock is smaller in the lagged information model than in the no information model because $(\omega + r)/(1+r) < A = (1-\theta+r)/(1+r)$[12]. This is because the relevant innovations that the consumer responds to differ in the two models. ν_{it} in the lagged information model only contains information on contemporaneous aggregate and individual shocks. η_{it} in the no information model also contains new information on lagged shocks.

Both the orthogonality failure and the smoothness result will still arise in the lagged information model, but their quantitative importance will differ.[13] Consider the regression of the change in consumption on the lagged income change in (12) again. The coefficient on lagged income will be

$$\beta = \frac{cov(\Delta c_t, \Delta y_{t-1})}{var(\Delta y_{t-1})}$$

$$= \frac{E\left\{\left[\frac{\omega+r}{1+r}\varepsilon_t + (1-\omega)\varepsilon_{t-1}\right]\varepsilon_{t-1}\right\}}{\sigma_\varepsilon^2} = 1-\omega \tag{22}$$

which is positive. Taking variances in (21) yields

$$\frac{\sigma_{\Delta c}}{\sigma_\varepsilon} = \sqrt{\left(\frac{\omega+r}{1+r}\right)^2 + (1-\omega)^2} \tag{23}$$

12 This follows from $\theta > 0$ and the relationship between θ and ω.

13 The test carried out by Campbell and Mankiw (1989) should not reject the model since their test only relies on instruments lagged at least two periods. Their rejection therefore is inconsistent with the model with lagged information.

which is less than one for small values of r.[14]

Which of the two models presented above is more reasonable? Ideally, one would consider a hybrid where agents obtain some noisy aggregate information with a lag. The two models can be thought of as special cases of this hybrid model which generates an ARMA(1,1) process for consumption changes. The predictions for β and the ratio of the variability of consumption to the variability of the income innovation lie between the predictions for the two polar cases considered above. I do not elaborate on this here because I have not found tractable generalizations to other income processes for the model with noisy signals on aggregate income.

Among the two polar models the one with lagged information seems better suited to explain the behavior of rational decision makers who form expectations on the basis of all available information since basic aggregate statistics are provided virtually for free by the news media. However, a rational agent will not only consider the costs, which are admittedly small, but also the benefits. Cochrane (1989) has shown that it is possible to calculate the loss from nonmaximizing behavior and found that these losses are generally small for small deviations from the optimal path. The same should be true here. I will present results on the utility loss from ignoring aggregate information in section 8 after showing what reasonable estimates for the individual and the aggregate income processes are. First, turn to the formulation of the model with more general income processes.

5. More General Income Processes

It is straightforward to extend the examples in the sections 3 and 4 to more general processes for income. First return to the version of the model with no information. Let the first differences in

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14 The implied VAR representation of the model is

$$\begin{bmatrix} \Delta y_t \\ s_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -(1-\omega) & (1+r) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ (1-k)\varepsilon_t \end{bmatrix}$$

where $k = (\omega + r)/(1+r)$ fitting the qualitative results of Campbell (1987).

individual income be stationary. This is a fairly general framework since it allows for stationarity in the levels as well, in this case the first differenced process has an MA unit root. Income consists of an aggregate and an individual component given by their respective Wold representations:

$$\Delta y_{it} = \phi(L)\varepsilon_t + \theta(L)u_{it} \tag{24}$$

$$\text{where} \quad \phi(z) = \sum_{i=0}^{\infty} \phi_i z^i$$

$$\theta(z) = \sum_{i=0}^{\infty} \theta_i z^i$$

Average per capita income is then given by

$$\Delta y_t = \phi(L)\varepsilon_t \tag{25}$$

Given stationarity, the process for individual income changes has a Wold representation

$$\Delta y_{it} = A(L)\eta_{it} \tag{26}$$

Individual consumption will follow

$$\Delta c_{it} = A\left(\frac{1}{1+r}\right)\eta_{it} \tag{27}$$

Define $\overline{\eta}_t$ as the mean of η_{it}. Equating (24) and (26) and summing over individuals yields

$$A(L)\overline{\eta}_t = \phi(L)\varepsilon_t \tag{28}$$

If $A(L)$ has no unit root (i.e. at least one of the two components is integrated of order one)[15] we can invert it to obtain

$$\Delta c_t = A\left(\frac{1}{1+r}\right)\overline{\eta}_t = A\left(\frac{1}{1+r}\right)A^{-1}(L)\phi(L)\varepsilon_t \tag{29}$$

15 The analysis proceeds analogously for stationary processes after canceling the common unit root in $\phi(L)$ and $A(L)$.

Under what conditions does (29) imply excess smoothness in a representative agent model for aggregate consumption? For small interest rates, a necessary and sufficient condition for excess smoothness is given by

$$\frac{1}{2\pi}\frac{f_A(0)}{f_\phi(0)}\int_{-\pi}^{\pi}\frac{f_\phi(\omega)}{f_A(\omega)}\,d\omega < 1 \tag{30}$$

where $f_x(\omega)$ is the normalized spectral density at frequency ω for process x. A derivation is given in Appendix A. Condition (30) shows that relative persistence of the component processes is important: The higher is the spectral density at frequency zero of aggregate income compared to the compound process (and thus compared to individual income) the more likely is the model to yield excess smoothness. But a second component is present in (30) indicating that the entire spectral shape of the processes also matters. This is the case because individuals use current period income changes to extract not only information on current income innovations but on the entire history as well. The relative dynamics of aggregate and individual income determine how agents evaluate an observed movement in income. Excess volatility of consumption can arise even if aggregate shocks are more permanent if certain spectral densities are not well represented in individual income.[16]

The examples in the previous sections demonstrated the orthogonality failure through the correlation at the first lag. For specific processes, this correlation can be recovered from (29). However, there is no obvious way to parameterize the occurrence of the orthogonality failure in general. Since Galí (1991) has shown that either excess smoothness or excess volatility has to imply the orthogonality failure I will not pursue this issue separately here and refer the reader to Galí for details.

Now turn to the model with lagged information. Rewrite (24) as

---

16 An example of such a case is an aggregate MA(1) in first differences with a coefficient of 0.3 combined with an individual MA(2) in first differences with coefficients 0.6 and -0.4 and an innovation variance ten times that of the aggregate income process. Aggregate income is more persistent, as measured by the spectral density at frequency zero. Nevertheless, aggregate consumption is more volatile than in the representative agent model.

$$\Delta y_{it} = \varepsilon_t + u_{it} + \overline{\phi}(L)\varepsilon_{t-1} + \overline{\theta}(L)u_{it-1} \tag{31}$$

$$\text{where} \quad \overline{\phi}(z) = \sum_{i=1}^{\infty} \phi_i z^i$$

$$\overline{\theta}(z) = \sum_{i=1}^{\infty} \theta_i z^i$$

Define v_{it} again as the contemporaneous innovation. Since all the previous values of the aggregate shocks can be observed and all the previous values of the individual shocks can be inferred we can again think of information consisting of the innovation v_{it} and the correction for the error made before. Equation (18) still defines the error made last period in attributing parts of the innovation to the aggregate and the individual processes. Analogously to equation (20) we obtain for the change in individual consumption

$$\Delta c_{it} = \left\{\phi\left(\frac{1}{1+r}\right)\omega + \theta\left(\frac{1}{1+r}\right)(1-\omega)\right\}v_{it} + (1+r)\left\{\phi\left(\frac{1}{1+r}\right) - \theta\left(\frac{1}{1+r}\right)\right\}\xi_{it-1} \tag{32}$$

Aggregating yields[17]

$$\Delta c_t = \left\{\phi\left(\frac{1}{1+r}\right)\omega + \theta\left(\frac{1}{1+r}\right)(1-\omega)\right\}\varepsilon_t + (1+r)\left\{\phi\left(\frac{1}{1+r}\right) - \theta\left(\frac{1}{1+r}\right)\right\}(1-\omega)\varepsilon_{t-1} \tag{33}$$

The regression coefficient of consumption changes on lagged income changes is given by

$$\beta = \frac{(1+r)\left\{\phi\left(\frac{1}{1+r}\right) - \theta\left(\frac{1}{1+r}\right)\right\}(1-\omega)}{\sum_{i=0}^{\infty} \phi_i^2} \tag{34}$$

As in the previous section, the orthogonality condition holds at all further lags because agents incorporate all aggregate information after one period. It is obvious that for small interest rates the condition $\phi(1) > \theta(1)$ is necessary and sufficient for a positive regression coefficient in (34). It turns out that the same condition together with invertibility of $\theta(z)$ is also sufficient for excess smoothness. A demonstration of this fact is given in Appendix A.

17 Equations (32) and (33) correspond to equations (11) and (12) in Goodfriend (1992).

In contrast to the no information model income dynamics do not play a role here. Only the relative persistence of aggregate and individual shocks as measured by $\phi(1)$ and $\theta(1)$ matter. This is because households can separate new information ν_{it} from lagged information which is not the case for the no information model.

6. Empirical Results on Micro Income Processes

The remainder of the paper explores whether the data bear out the implications of the models studied above. The strategy I pursue is to estimate simple models for the micro and macro income processes first. Using these estimates I calculate the implied values of the excess smoothness ratio and the regression coefficient for the orthogonality test at the aggregate level. The results are then easily compared to the aggregate sample values of these statistics.

I start in this section by presenting results on individual income processes. Previous studies in this area reveal that income innovations for individuals are less persistent than shocks to aggregate income and that individual income variation is far more important.

MaCurdy (1982) and Abowd and Card (1989) have analyzed the time series structure of earnings in micro data. They find that the log of earnings changes for male household heads in the U.S. is well described by an MA(2). Both MA coefficients are negative, with the first one between -0.25 and -0.4 and the second one closer to zero. The variance of log earnings changes is substantial. The standard deviations range from about 0.25 to a high of 0.45 for certain years. This means that a one standard deviation change in earnings is 25 percent to 45 percent of the previous level. Individual income risk is clearly the main source of income uncertainty individuals face.

MaCurdy only analyzes data from the Panel Study of Income Dynamics which is conducted annually. Abowd and Card also present results for data from the control groups of the Denver and Seattle Income Maintenance Experiments which correspond to semiannual income. They find generally

first order autocorrelations that are even more negative for these data. However, this may not result from the different sampling frequency but from the fact that the experiment oversampled relatively poor households.

While these studies refer to earnings, results for the (annual) family income process are provided by Hall and Mishkin (1982). They estimate a restricted MA(3) for income changes with results very similar to the studies mentioned above. Family income apparently follows a process very similar to individual earnings.

None of these results are directly suited for the present purpose. The stylized facts on aggregate consumption have all been established on quarterly series. In order to have analogous results for individual income I estimated restricted covariance models with quarterly data that I constructed from the 1984 Survey of Income and Program Participation (SIPP). This panel survey was conducted three times a year from late 1983 to the beginning of 1986 in about 20,000 households and collected monthly income information. The interviews took place on a rolling basis, with one fourth of the sample being interviewed each month. In each interview, information was collection on the four past months. From these data I constructed a panel of quarterly income from the fourth quarter of 1983 to the first quarter of 1986, the longest span for which information on the entire sample is available.

Consumption decisions are most likely made at the family level. I therefore selected families that can be followed continuously throughout the sample period and did not change head or spouse. Most likely, events that change household composition in a major way will also lead to large income changes. The sample selection will therefore tend to understate the variance of income changes. Furthermore, I limited the sample to households whose head did not go to school in any part of the sample period. The latter group may have large movements in income which are anticipated by the individuals but would appear as random elements in the estimation. For example, an individual just finishing school will have a large increase in income. But this jump will have been foreseen and has therefore, according to the model, already been incorporated in previous consumption decisions. I also eliminated non-family households since I cannot judge whether they make joint

or individual consumption decisions. Finally, I limited the sample to families with heads between the ages of 16 and 70 during the survey period. Appendix B contains further details on the construction of the sample.

Table 1

Basic Sample Statistics

	SIPP Sample		CPS Sample	
	Mean	Std. Dev.	Mean	Std. Dev.
Age	43.9	12.9	42.5	13.4
Years of Schooling	12.6	3.25	12.5	3.22
Non-White	0.12	0.32	0.13	0.34
Male	0.77	0.42	0.73	0.44
Never Married	0.09	0.29	0.14	0.35
Family Size	3.03	1.50	2.82	1.56
Family Income 1984 [quarterly]	6,663	4,933	6,666	5,060
Sample Size	8,176		25,033	

The correct income concept is net family income from all sources excluding capital income. Variables on total family income and income from capital are provided on the SIPP user tapes; these are aggregated from an array of detailed questions on various income categories for each family member. I use these variables although there are some problems associated with them. First, tax information is only collected infrequently and cannot be apportioned to single months. This is probably the most severe shortcoming of the data because gross income will have a higher variance and (in a progressive tax system) exhibit more transitory fluctuations. Furthermore, the individual variables that make up family income can have imputations. Since the imputations occur at the disaggregated level it would be rather arbitrary to decide which observations to delete because of the imputations. I decided to use all the data. Imputations should lower the variance of income

changes, presumably largely at the cost of the transitory income component.[18] Finally, all disaggregated income items are topcoded at $8,333 per month. It is impossible to decide from the aggregated income items which variables have been topcoded. The topcoding should only affect a small portion of the sample and will also reduce the income variance.[19]

I provide some basic characteristics of the sample in table 1 which also presents results from the March 1985 Current Population Survey. In most respects the SIPP sample matches the general population very closely.

Measurement error. Before turning to the quarterly estimates of income dynamics I will consider some adjustments that seem sensible in the presence of reporting error in earnings. Absent any validation information, the true income process can generally not be recovered from covariance estimates of survey income reports if arbitrary measurement error is present in the data.[20]

The main measurement problem in the SIPP seems to be related to the timing of changes. As a referee pointed out, family income in the SIPP has the feature that it is constant over a period of time and only changes at infrequent intervals. This constancy of income in the SIPP is mainly a feature of the interview structure. Recall that information is collected retrospectively by asking respondents separately about each of the past four months. 47 percent of the families in the sample report no change in income from one month to the next if the information was collected in the *same interview.* Only 9 percent report constant income in two adjacent months if these reports come from *different interviews.* A large fraction, 27 percent, of respondents reports constant income within the entire interview.

18 Coder (1992) finds that imputations lower the cross-sectional variance of the levels of income.

19 About 2 percent of the households in each wave report total income of $ 8,333 or more. This is an upper bound for the incidence of topcoding since it may result by summing various components that may each be below the cutoff. Deleting all the households that have income above this level in at least one months during the sample period reduces the sample size by 12 percent. The variance of income changes is reduced to 40 percent in this sample but the autocorrelations are very similar.

20 As far as I know, the only study of this type that looks explicitly at income is the exact match between federal tax records and the SIPP undertaken by the Census Bureau. Coder (1992) compares the tax records with annual survey income (adding up 12 monthly reports) from the 1990 SIPP and finds that respondents tend to understate their true income except in the lowest two income deciles. His findings indicate that there must be a substantial negative correlation between survey income and the measurement error when treating the tax records as true income.

This pattern is typically referred to as "seam bias" and is observed for most variables in the SIPP. Despite considerable research efforts at the Census Bureau the nature of the seam bias is not fully understood (Jabine, 1990). There seems to be some consensus that most respondents tend to report correct transitions (into and out of program participation, of income recipiency, and of amounts) but fail to identify the exact timing of transitions or changes. Changes during the four months of an interview seem to be underreported while too many changes are reported at the seams between interviews.

Similar patterns were found by Goudreau et.al. (1984) in a study using data from the Income Survey Development Program (ISDP). They compare interview reports of AFDC receipts for each of the past three months with administrative records. They find that more than half the respondents reported accurate amounts. However, the vast majority of these respondents received constant amounts. Half of the remaining respondents reported accurate amounts for some months. 27 percent of them reported the most recent payment for an earlier month and 24 percent missed the timing of a change by one month. The remaining respondents reported incorrect amounts for all months; 62 percent of them reported a multiple or fraction of the truth.

I present two very simple measurement error models which capture some of the features found in these validation studies. First, consider a classical measurement error that is constant for an entire interview so that respondents under- or overreport income by the same amount for all four months of the interview.[21] Denoting true income as y^*_{ijt} and the measurement error by μ_{ij} monthly income can be written as

$$y_{ijt} = y^*_{ijt} + \mu_{ij} \tag{35}$$

Subscripts *i* refer to families, *j* to interviews, and *t* to months. The measurement error μ_{ij} is assumed to be serially uncorrelated and uncorrelated with true income. This model will tend to overstate the importance of the measurement error if measurement error is actually negatively correlated with true income.

21 I focus on such an interview specific measurement error since it can be identified from the data using the interview structure of the SIPP. This is not true for a measurement error that is variable from month to month.

The second model posits that respondents always report the true income they received in the last month of the interview for every month in that interview. Hence, according to this model, and somewhat counterfactually, there should be no changes in income within any interview.

Both measurement error models generate a seam pattern but neither of them is plausible as a sole explanation of all the features of the reporting error. Nevertheless, each model captures one of the features found in the Goudreau et.al. (1984) study. Furthermore, these two models have very different implications for the final estimates of income dynamics. If some respondents behave according to either model then they will provide reasonable bounds for the true income dynamics.

I proceed in the following way. I first present a few features of the monthly income data. These will allow me to give numerical estimates to the key parameters of the measurement error models. Given their simple structure it is straightforward to calculate their implications for quarterly data. Both models imply that the measurement error in quarterly income changes follows an MA(2). After estimating the covariance structure of the quarterly income changes I can adjust the covariance estimates for measurement error to obtain parameters for the "true" implied income processes.

The following results for the monthly data for my SIPP sample are based on the first eight waves.[22] The variance of income changes (dividing income by 1,000) is 2.44 for income changes between the first month of an interview and the last month of the previous interview. For income changes within a single interview the corresponding variance is 1.12, less than half of the on-seam variance. The first autocovariances of income changes vary between -0.68 and -0.45 for various months in the interview, indicating at best small differences within the interview as well as across the seam. All other autocovariances are small and insignificant, except for the fourth autocovariance for changes on the seam, which is -0.50.

These covariances reveal a number of interesting patterns. First, most of the variance within the interview is completely transitory while some of the variance at the seam is permanent. Furthermore, the on-seam pattern is consistent with both measurement error models. The relevant parameter in

22 Only two of the four rotation groups in the 1984 SIPP had nine interviews.

the model of classical measurement error which is constant within the interview is σ_μ^2. It can be estimated as half the difference between the on seam-variance (2.44) and the off-seam variance (1.12) as is easily seen by differencing (35)

$$\Delta y_{ijt} = \begin{cases} \Delta y_{ijt}^* & \text{within interview} \\ \Delta y_{ijt}^* + \Delta\mu_{ij} & \text{across interviews} \end{cases} \tag{36}$$

This yields a value of 0.66 for the measurement error variance. Another estimate is obtained as minus the fourth on-seam autocovariance which is 0.50. Optimally combining the sample information results in an estimate of 0.59 with a standard error of 0.024.[23]

The parameters of the measurement error model where respondents report just the income amount of the most recent month depend on the true income process. The monthly data seem to suggest a random walk plus transitory noise as a reasonable approximation to the true income process. I will calculate the bias for this case, without restricting the later quarterly estimates to this simple model. Since this model only allows changes at the seam I will use the seam changes in estimating the model parameters. Writing $\Delta y_{it} = \varepsilon_{it} + \Delta u_{it}$ for the income process the on-seam variance is $4\sigma_\varepsilon^2 + 2\sigma_u^2$ and the fourth autocovariance is $-\sigma_u^2$. The two moments just identify the parameters at $\sigma_u^2 = 0.50$ and $\sigma_\varepsilon^2 = 0.36$.

Due to the fact that the SIPP interviews cover four months, the measurement error will follow an MA(2) at the quarterly level for the first model. For the second model it is not strictly correct to talk of a measurement error process but an equivalent adjustment to the variance and first two autocovariances can be obtained. Calculations accounting for the overlapping rotation group design of the data are given in Appendix C. Before presenting the results below in table 4 I will turn to measured quarterly income.

23 Formally, the overidentifying restrictions implied by this simple model for the measurement error are rejected by the data. On the one hand, the covariances for single months are estimated rather precisely due to the large sample size. On the other hand, there are other implications of the data that are neglected here. For example, the variance of monthly income changes increases towards the end of the interview, maybe indicating better recall of changes in the income stream for the more recent months.

Table 2

Covariance Matrix of Income Changes

(Income / 1000)

(standard errors in parentheses)

	84:1	84:2	84:3	84:4	85:1	85:2	85:3	85:4	86:1
84:1	10.321 (0.763)	-0.254	-0.126	-0.101	0.047	-0.039	-0.026	0.006	0.044
84:2	-2.390 (0.362)	8.592 (0.507)	-0.290	-0.168	-0.001	0.040	0.013	-0.039	-0.023
84:3	-1.207 (0.345)	-2.538 (0.406)	8.937 (0.625)	-0.236	-0.197	0.002	0.036	-0.064	-0.002
84:4	-1.023 (0.329)	-1.554 (0.331)	-2.233 (0.357)	9.978 (0.687)	-0.355	-0.142	-0.080	0.103	-0.026
85:1	0.510 (0.331)	-0.009 (0.304)	-1.971 (0.290)	-3.758 (0.554)	11.249 (0.720)	-0.306	-0.132	-0.036	0.058
85:2	-0.369 (0.228)	0.350 (0.237)	0.021 (0.216)	-1.332 (0.222)	-3.044 (0.354)	8.792 (0.461)	-0.245	-0.188	-0.013
85:3	-0.247 (0.213)	0.112 (0.201)	0.321 (0.233)	-0.755 (0.219)	-1.322 (0.249)	-2.175 (0.286)	8.954 (0.462)	-0.259	-0.171
85:4	0.068 (0.240)	-0.376 (0.200)	-0.621 (0.211)	1.066 (0.242)	-0.395 (0.289)	-1.815 (0.244)	-2.528 (0.295)	10.641 (0.631)	-0.326
86:1	0.472 (0.263)	-0.219 (0.214)	-0.024 (0.274)	-0.269 (0.246)	0.647 (0.283)	-0.124 (0.257)	-1.692 (0.241)	-3.511 (0.497)	10.884 (0.717)

Covariances below the diagonal, correlations above the diagonal

The dynamics of measured income. Measured family income is aggregated into quarterly amounts. The estimation of the quarterly income process proceeds in three further stages. In a first step, I regressed changes in family income on a constant, changes in total family size, changes in the number of children, and age of the head to eliminate deterministic components of income dynamics; these regressors are similar to the ones used by Hall and Mishkin (1982). Separate regressions were run for each quarter. Thus the data will be purged of all common seasonal and aggregate components as well. None of the regressors explains income changes very well; as is usual in such regressions

the R^2s range from only 0.002 to 0.008! Adding lagged labor market indicators, like number of earners in the household, weeks worked by the head, weekly hours, occupation, and industry as additional regressors hardly changes the results.

The second step was to estimate the unrestricted covariance matrix of residual income changes. Table 2 displays this 9 x 9 matrix. The standard deviations of quarterly family income changes range from \$2,931 to \$3,353. The mean level of family income is \$7,278. The standard deviations are between 40 and 46 percent of the income level, somewhat higher than MaCurdy's and Abowd and Card's findings on annual data.

The first column in table 3 presents minimum distance estimates where the diagonals of the above covariance matrix are restricted to have constant elements.[24] The first two autocorrelations are large in absolute value and comparable to the estimates for annual earnings. Since time aggregation of ARMA processes does not have this feature measurement error may be responsible for this finding. Beyond the second lag, the autocorrelations are closer to zero but some are still significant. The positive values at the 4th and 8th lag stick out. These may indicate that there are seasonal components at the individual level in these data. A look at table 2 shows that the 4th order autocorrelation is particularly large at for the 4th quarter. Differing seasonal employment patterns in the last quarter, e.g. in construction and retail trade, may be an explanation. However, including lagged industry and occupation dummies in the first stage regressions changes the results little.

The specification test at the bottom of table 3 also reveals that the data is not very happy with the stationarity restrictions; there are significant differences in the variances and autocorrelations over the year. Income changes are less variable in summer as can be seen in table 2. These findings are indicative of possible deterministic components in household income changes, i.e. changes that occur with some regularity but not in the same direction for every household. Compared to the short term dynamics in income changes as captured in the first two autocorrelations these regularities do not seem overly large. Lacking any identifying information on deterministic income changes

24 I initially estimated covariances. The standard errors on the reported autocorrelations are obtained by the delta method.

Table 3

Stationary Processes for Income Changes

(standard errors in parentheses)

Coefficient	stationary process	MA(3)	MA(2)
Standard Deviation	2951 (45.5)	2900 (44.4)	2893 (23.6)
1st autocorrelation	-0.274 (0.009)	-0.271 (0.009)	-0.270 (0.009)
2nd autocorrelation	-0.169 (0.012)	-0.162 (0.012)	-0.182 (0.010)
3rd autocorrelation	-0.042 (0.012)	-0.025 (0.010)	---
4th autocorrelation	0.058 (0.013)	---	---
5th autocorrelation	-0.019 (0.012)	---	---
6th autocorrelation	-0.029 (0.014)	---	---
7th autocorrelation	-0.007 (0.017)	---	---
8th autocorrelation	0.046 (0.026)	---	---
Specification test χ^2-statistic [dof] p-value	60.2 [36] 0.007	82.8 [41] 0.000	89.4 [42] 0.000
Test for Stationarity χ^2-statistic [dof] p-value	---	38.3 [26] 0.056	30.8 [21] 0.077

and for reasons of tractability I will work with a stationary MA(2) model for income changes. The test in the last row of table 3 indicates that stationarity is not as big a problem once higher order autocorrelations are restricted to zero.

The micro income process. Table 4 presents adjusted estimates for the MA(2) income process based on the results the measurement error models discussed previously. Using the results in the last

column of table 3 and subtracting the variation due to constant-within-interview measurement error yields a standard deviation of "true" income changes of $1,743. This implies a ratio of true variance to total variance of 0.36, a value substantial below the finding of about 0.65 or better reported by Bound et.al. (1993) from various validation studies for annual earnings. For the second measurement error model based on reported amounts from the most recent month the standard deviation of implied income changes is much higher at $2,619 implying a ratio true to total variance of 0.82. I adjust these standard deviation of family income changes by the average of the CPI for urban consumers (base 1982-84) over the sample period (which is 105.3). This yields values of $1,655 and $2,487 which should be compared to a level of real quarterly family income of $6,902 in these data.

Table 4

Measured Income, Measurement Error and "True" Income

	Variance (Std. Dev.)	1st autocovariance (autocorrelation)	2nd autocovariance (autocorrelation)
Measured Income Changes	8.37 (2893)	-2.26 (-0.270)	-1.52 (-0.182)
Measurement Error Model 1	5.33 (2309)	-1.78	-1.48
Implied Income Changes for Model 1	3.04 (1743)	-0.48 (-0.158)	-0.04 (-0.013)
Measurement Error Model 2	1.50 (1225)	0.50	-1.62
Implied Income Changes for Model 2	6.86 (2619)	-2.76 (-0.402)	0.10 (0.015)

Making the appropriate adjustments for the first measurement error model in the first and second autocorrelations yields values of -0.16 and -0.01, respectively. Practically all the second order autocorrelation is completely due to measurement error. This is also true for the second model of measurement error but here the first autocorrelation is more negative than in the measured data and has a value of -0.40. The first measurement error model clearly removes a large fraction of the transitory variation in the data while the second model actually adds transitory elements. As I have

pointed out above, heterogeneity in the individual income process and income fluctuations known to the individual may bias these estimates. I present evidence below that this does not affect the conclusions very much as far as it leads to an overestimate of the individual income variance while the results are less robust to changes in the autocorrelations.

7. Aggregate Stylized Facts on Income and Consumption

In this section I report the stylized facts pertaining to income and consumption processes in aggregate data. This has two purposes. First, I will try to establish some simple time series model for the aggregate income process. Together with the results of the previous section this will allow me to calculate predictions from the model with heterogeneous agents for aggregate consumption. I will therefore also report results on consumption here to compare them to the predictions in the following section.

In order to replicate the results often cited in the literature I make the same adjustments to the NIPA data as Blinder and Deaton (1985) did.[25] However, since the micro level estimates in the previous section are for households rather than individuals all macro series used are also on a per household basis rather than on a per capita basis.[26] My sample ranges from the first quarter of 1954 to the fourth quarter of 1990, the data are taken from the 1991 Citibase tape. All variables are in levels not in logs.[27] A detailed description of the adjustments I make is given in Appendix B.

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25 Unlike Blinder and Deaton (1985) I did not adjust income and consumption for nontax payments to state and local governments since the series on Citibase is only available starting in 1958. For the post-1958 sample the difference is completely inconsequential.

26 Since no quarterly series of the number of households is available for the sample period I linearly interpolated annual estimates of average household size.

27 Typically logs of variables are preferred to levels on the grounds that the level variables exhibit growing variances over time. Regressing squared changes of the variables or squared residuals from the models in table 5 and 6 on a linear trend I found no evidence of this in these data.

Sample Period	AR(1)	MA(2)		
		First coefficient	Second coefficient	Std. Dev. of Income Innovations
NIPA 1954-1984	0.346 (0.085)	0.375 (0.090)	0.013 (0.090)	47.8 (3.03)
NIPA 1954-1990	0.288 (0.080)	0.299 (0.083)	0.010 (0.083)	49.0 (2.85)

Table 5

Aggregate Stylized Facts on First Differences of Income

(standard errors in parentheses)

Table 5 presents results on the income process. The income series refers to "labor" income, i.e. disposable income excluding capital income. There is a slight conceptual difference to the micro estimates since the aggregate income series excludes taxes. However, whether taxes are excluded or not makes little difference for the aggregate estimates. I therefore use the series commonly used in the literature. As for individual income I will use an MA(2) model for the first differences of aggregate income but I also present results for an AR(1). The MA coefficients are estimated by conditional least squares,[28] the AR model is estimated by OLS. I report results for two different sample periods. 1954 to 1984 is the period of the Binder and Deaton (1985) dataset that has been used extensively by various researchers. Notice that extending the sample to 1990 reduces the autocorrelation in the income changes slightly. Both the AR(1) and the MA(2) fit the data well. The quarterly standard deviation for aggregate per household income is only around $50, compared to the $1,600 or more I found for the individual income component above!

Table 6 reports some results on aggregate consumption for similar sample periods as the previous table. It has been customary in the macro literature to use consumer expenditure on nondurables and services as consumption measure. Like Blinder and Deaton I eliminated expenditures on

28 This ignores the fact that initial values are assumed rather than derived from data when filtering the process for the MA innovations.

clothing and shoes from the nondurable consumption series. To make units comparable to total income I multiplied these expenditures by the sample average of the ratio of total expenditures to expenditures on nondurables and services.

Table 6

Aggregate Stylized Facts on First Differences of Consumption

(standard errors in parentheses)

Sample Period	Coef. of Consumption Changes on Income Lag	AR (1) coefficient	MA (1) coefficient	Excess Smoothness Ratio
1954-1984	0.121 (0.049)	0.210 (0.088)	0.197 (0.088)	0.601 (0.064)
1954-1990	0.110 (0.045)	0.200 (0.082)	0.206 (0.081)	0.578 (0.055)

The table reports the regression coefficient of consumption changes on lagged income changes which is in the order of 0.11 and significant. Consumption changes are positively autocorrelated as measured by an AR(1) or MA(1) parameter. The last column gives the excess smoothness ratio of about 0.6. All these estimates are in line with previous findings in the literature.

8. Predictions from the Model

I am now ready to present predictions from the models using the empirical estimates for the individual and aggregate parts of the income process. Since the estimates vary slightly for different sample periods and for the two measurement error models I will present a number of results. This will also serve as a robustness check.

I assume that both the individual income process and the aggregate income process are described by an MA(2) in first differences.

$$\Delta y_{it} = (1+\phi_1 L+\phi_2 L^2)\varepsilon_t + (1-\alpha_1 L-\alpha_2 L^2)u_{it}$$
$$= (1-\theta_1 L-\theta_2 L^2)\eta_{it} \quad (37)$$

The consumption processes for the two models are given in (29) and (33), respectively. In the case of the no information model aggregate consumption follows an ARIMA(2,1,2) process. For the lagged information model, consumption changes are an MA(1). Appendix A presents the formula for β the coefficient for a regression of consumption changes on lagged income changes, for the no information model and the excess smoothness ratio $\sigma_{\Delta c}/\sigma_\varepsilon$.

Predictions for these parameters are shown in table 7 and compared to the aggregate stylized facts about consumption from table 6. The base case uses the estimates for the individual income process unadjusted for measurement error and the 1954 - 1990 results for aggregate income. The full information representative agent model implies a β of zero in this case and $\sigma_{\Delta c}/\sigma_\varepsilon$ of 1.31. Both the no information model and the lagged information model predict parameters which are very different from this benchmark and which are qualitatively in the direction of the actual aggregate estimates. The results for no information model are superior to the lagged information model in the base case. Still, both models considerably overpredict β and the lagged information model overpredicts $\sigma_{\Delta c}/\sigma_\varepsilon$ by about a factor of two.

The last column presents the utility loss for a household that uses no aggregate information compared to the full information case.[29] The loss is expressed in Dollars per quarter and household and is calculated for a coefficient of relative risk aversion of two. It amounts to 1.43 Dollars or 0.02 percent of total utility. This is similar to the findings by Cochrane (1989) who estimated the utility loss for a representative consumer exhibiting excess sensitivity. The loss for higher risk aversion is easily obtained by dividing by two and multiplying by the new coefficient. Even for a risk aversion

29 Instead of comparing the model with no information to the Goodfriend model I use a model with full contemporaneous information on aggregate variables as benchmark. Utility for this model is calculated much more easily than for the lagged information model. The utility differences I present are therefore upper bounds for the differences between the two models in the paper.

Table 7

Comparison of Model Predictions and Aggregate Estimates

Case	Aggregate Estimates		No Information Model		Lagged Information Model		Utility Loss
	β	$\sigma_{\Delta c}/\sigma_\varepsilon$	β	$\sigma_{\Delta c}/\sigma_\varepsilon$	β	$\sigma_{\Delta c}/\sigma_\varepsilon$	[$/quarter]
1	0.110	0.578	0.292	0.523	0.883	1.026	1.425
2	0.121	0.601	0.319	0.548	0.912	1.099	1.569
3	0.110	0.578	0.374	0.913	0.447	0.958	0.173
4	0.110	0.578	0.443	0.717	0.716	0.946	0.536
5	0.110	0.578	0.293	0.525	0.882	1.025	1.401

base case: σ_u = $2,471, $\alpha_1 = 0.431$, $\alpha_2 = 0.225$, σ_ε = $49.0, $\phi_1 = 0.300$, $\phi_2 = 0.010$,
interest rate = 0.01, mean income = $6,902, coef. of rel. risk aversion = 2
Case 2: As base case but σ_ε = $47.8, $\phi_1 = 0.375$, $\phi_2 = 0.013$
Case 3: As base case but σ_u = $1,633, $\alpha_1 = 0.165$, $\alpha_2 = 0.013$
Case 4: As base case but σ_u = $2,292, $\alpha_1 = 0.489$, $\alpha_2 = -0.019$
Case 5: As base case but σ_u = $1,236

coefficient of 10 the loss would still be minor. This provides some evidence that the assumptions of the no information model seem to be quite reasonable: it does not pay to collect aggregate information to improve consumption decisions.

The next rows present variations on the base case. Case 2 uses the aggregate estimates for the 1954 - 1984 period; the results are very similar. Case 3 presents calculations with the micro income process with the adjustment for measurement error according to the first model. In this model of an interview specific measurement error most of the transitory variation is removed from the micro income estimates. The results in this case are much less favorable to the no information model while the lagged information model improves. Case 4 presents the results for the second measurement error model where respondents report true income of the previous month for the entire SIPP interview. Despite the fact that micro income still contains a large transitory component, the results are very similar to the ones obtained with the first measurement error model, in particular for the no information model. Further calculations, which are not reported, showed that removing

the MA(2) term from measured income is responsible for the differences to the base case. This does not mean that the size of the coefficient α_1 does not affect the predictions of the model. Rather, the predictions are relatively insensitive in the particular range implied for α_2 by the two alternative measurement error models. Case 5 illustrates that the estimate of the micro variance, on the other hand, has little impact on the results. In this case, the micro variance is set to one fourth of the value in the base case. The results are completely unaffected.

Given the similar conclusions from both measurement error models, it is unlikely that the results will depend strongly on the exact response behavior of households in the SIPP. Since these results only pertain to the most simple minded version of a life-cycle consumption model it is not surprising that they do not match the data more closely. But it becomes clear that incomplete information may play an important role in explaining the orthogonality failure and excess smoothness at the aggregate level.

9. Concluding Comments

In this paper I have analyzed the implications of heterogeneity in income and incomplete information on the source of income shocks for the form of the aggregate consumption process and its relation to observed income. The failures of the full information life-cycle consumption model usually found in aggregate data clearly arise if individual consumers adjust their consumption correctly to individual income innovations but do not care to distinguish aggregate and idiosyncratic income variation. Using estimated parameter values for individual and aggregate income processes, the model gives predictions that deviate substantially from the full information benchmark. However, the results indicate too much correlation of consumption changes with lagged income but not smooth enough consumption. Nevertheless, heterogeneity in income and incomplete information seem to account for a large portion of the deviations from the full information case.

Rational expectations models with incomplete aggregate information have mostly used the assumption that aggregate information arrives with a one period lag. In the present context, the no information model seems to yield somewhat better results than the lagged information model but does not clearly dominate it. Some combination of the two models seem more reasonable as a description of reality. Consumers may not deliberately collect aggregate information. But their interaction with many other individuals will reveal a lot to them about the nature of their own income process. Formalizing models in which aggregate information arrives more slowly should be an area that deserves more attention.

The feature that drives the results in this paper is that the model yields an autocorrelated process for aggregate consumption changes. Galí (1991) has shown that excess smoothness of consumption can be characterized in the frequency domain with less restrictive assumptions than in Deaton (1987) or Campbell and Deaton (1989). Essentially, his results stem from the autocorrelation in consumption changes and are therefore consistent with the predictions from the no information model.

A number of other models have been suggested that lead to autocorrelated consumption. A simple model of habit formation (Deaton, 1987) or slow adjustment of consumers to income shocks (Attfield, Demery, and Duck, 1992) also leads to an AR(1) for consumption changes. Unlike for the models studied here, the micro parameters are generally not estimable in these cases so the models cannot be subjected to the same stringent test. Furthermore, these models imply that consumption should have the same autocorrelation structure in micro and in aggregate data. This seems to be at odds with the empirical findings.

Although in this paper I have focussed on implications of the no information model for aggregate data the model is roughly consistent with previous findings on micro data for consumption. It predicts correctly that the orthogonality conditions should not be rejected in panel data. The approach taken by Altonji and Siow (1987), Zeldes (1989) and Runkle (1991) is consistent with the model presented here. These studies find little evidence against the permanent income model with food consumption data from the PSID. The exception is Zeldes (1989), who finds some evidence for such correlations for low wealth consumers in the PSID, interpreting them as liquidity constraints.

It seems quite reasonably a priori that part of the population is liquidity constraint. Interactions of liquidity constraints and precautionary savings motives with the incomplete information assumption are considered in Deaton (1991). In numerical simulations Deaton finds a regression coefficient of consumption growth on lagged income growth of 0.42 and a smoothness ratio just below one. His results are for logs of the variables and are therefore not directly comparable to mine. Nevertheless, it seems that incomplete information may be the major factor driving these results.

Since the specifications in this paper are very restrictive future research should incorporate incomplete information into more sophisticated models. Finite lifetimes and superior information of consumers about income changes are possible candidates that may play an important role in bringing the results presented here better in line with the data.

Appendix A

Derivation of Conditions for Excess Smoothness

Let $\beta \equiv 1/(1+r)$ so that excess smoothness in the aggregate is given by $\sigma^2_{\Delta c} < \phi^2(\beta)\sigma^2_\varepsilon$ or

$$\Psi \equiv \frac{\sigma^2_{\Delta c}}{\phi^2(\beta)\sigma^2_\varepsilon} < 1 \tag{A1}$$

Consider the no information case. Using (29) in the text the spectral density of aggregate consumption changes is

$$h_{\Delta c}(\omega) = \frac{A^2(\beta)}{2\pi} \frac{|\phi(e^{-i\omega})|^2}{|A(e^{-i\omega})|^2} \sigma^2_\varepsilon \tag{A2}$$

The variance of consumption changes can be found by integrating (A2)

$$\sigma^2_{\Delta c} = \int_{-\pi}^{\pi} h_{\Delta c}(\omega)\, d\omega = \int_{-\pi}^{\pi} \frac{A^2(\beta)}{2\pi} \frac{|\phi(e^{-i\omega})|^2}{|A(e^{-i\omega})|^2} \sigma^2_\varepsilon\, d\omega \tag{A3}$$

so that the quantity Ψ is given by

$$\begin{aligned}\Psi &= \frac{1}{2\pi}\frac{A^2(\beta)}{\phi^2(\beta)} \int_{-\pi}^{\pi} \frac{|\phi(e^{-i\omega})|^2}{|A(e^{-i\omega})|^2}\, d\omega \\ &= \frac{1}{2\pi}\frac{A^2(\beta)}{\phi^2(\beta)} \frac{\sigma^2_\eta}{\sigma^2_\varepsilon} \int_{-\pi}^{\pi} \frac{h_\phi(\omega)}{h_A(\omega)}\, d\omega \\ &= \frac{1}{2\pi}\frac{A^2(\beta)}{\phi^2(\beta)} \frac{\sigma^2_\eta}{\sigma^2_\varepsilon} \frac{\sigma^2_{\Delta y}}{\sigma^2_{\Delta y i}} \int_{-\pi}^{\pi} \frac{f_\phi(\omega)}{f_A(\omega)}\, d\omega\end{aligned} \tag{A4}$$

where $f_x(\omega) = h_x(\omega)/\sigma^2_x$ is the normalized spectral density of process x. Taking limits as the interest rate approaches zero gives the following expression which appears as (30) in the text:

$$\lim_{r\to 0} \Psi = \frac{1}{2\pi}\frac{f_A(0)}{f_\phi(0)} \int_{-\pi}^{\pi} \frac{f_\phi(\omega)}{f_A(\omega)}\, d\omega \tag{A5}$$

Now turn to the lagged information model. From (33)

$$\frac{\sigma^2_{\Delta c}}{\sigma^2_\varepsilon} = [\phi(\beta)\omega + \theta(\beta)(1-\omega)]^2 + (1+r)^2[\phi(\beta)-\theta(\beta)]^2(1-\omega)^2 \tag{A6}$$

Using condition (A1) and letting interest rates get small we obtain

$$\lim_{r \to 0} \Psi < 1$$

$$\Rightarrow \; [\phi(1)\omega + \theta(1)(1-\omega)]^2 + [\phi(1) - \theta(1)]^2 (1-\omega)^2 \; < \; \phi^2(1) \tag{A7}$$

Define $K(\omega) \equiv [\phi(1) - \theta(1)]\omega + \theta(1)$ which will be positive given $\phi(1) > \theta(1) > 0$. The latter inequality holds if $\theta(z)$ is invertible. Notice that (A7) can be rewritten as

$$K^2(\omega) \; + \; [\phi(1) - K(\omega)]^2 \; < \; \phi^2(1) \tag{A8}$$

Thus we have to show that (A8) is satisfied. Use $\phi(1) > \theta(1)$, multiply both sides by $1-\omega$ and rearrange to get

$$K(\omega) \; = \; [\phi(1) - \theta(1)]\omega + \theta(1) \; < \; \phi(1) \tag{A9}$$

Recall that $K(\omega)$ is positive, multiply both sides of (A9) by twice $K(\omega)$ and add $\phi(1)^2$ to complete the square. Rearranging yields (A8) which completes the proof.

Empirical Formulation. In the empirical model in section 8 both the aggregate and the individual income component are described by an MA(2). Then $A(L) \; = \; 1 + a_1 L + a_2 L^2$. The roots of this polynomial are defined by $\mu^2 + a_1\mu + a_2 \; = \; 0$. Writing consumption changes in its series representation.

$$\Delta c_t \; = \; \frac{A\left(\frac{1}{1+r}\right)}{\mu_1 - \mu_2} \sum_{i=0}^{\infty} (\mu_1^{i+1} - \mu_2^{i+1})(\varepsilon_{t-i} + \phi_1 \varepsilon_{t-1-i} + \phi_2 \varepsilon_{t-2-i}) \tag{A10}$$

This can be used to derive the regression coefficient of consumption changes on lagged income changes

$$\beta \; = \; \frac{A\left(\frac{1}{1+r}\right)}{(\mu_1 - \mu_2)(1 + \phi_1^2 + \phi_1^2)} \{(\mu_1 - \mu_2 + \mu_1^3 - \mu_2^3)(\phi_1 + \phi_1\phi_2) + (\mu_1^2 - \mu_2^2)(1 + \phi_1^2 + \phi_2^2) + (\mu_1^4 - \mu_2^4)\phi_2\} \tag{A11}$$

The variance of consumption changes can either be found by solving (A3) for the relevant processes or by solving the Yule-Walker equations corresponding to the ARMA(2,2) given by (A10). I have done the latter numerically.

Appendix B

Sample Selection and Variable Definitions

Construction of the SIPP Sample. The 1984 Survey of Income and Program Participation was conducted in nine interview waves. Households were interviewed on a rolling basis, starting October 1983 for the first rotation group and ending July 1986 with the last rotation group. For wave 2 rotation group 2 was not interviewed, for wave 8 there is no interview for rotation group 3. In each interview, questions were asked about income for each of the previous four month. Thus monthly income data are available for all rotation groups from September 83 to March 86. Since I intend to construct quarterly observations I started with the October 83 variables.

I started by matching household heads from the nine interview waves. This resulted in 12,874 matches. I then restrict the matched sample as described in the text by selecting continuous heads for the period of analysis, that did not change marital status or their level of schooling in any month. Per capita family income is constructed by subtracting property income (F*-PROP) from total family income (F*TOTINC) and deflating by the monthly CPI for urban consumers (1982-1984 base). Finally, I corrected reported age of the head so that age increments by one every four quarters. The final sample contains quarterly variables from the last quarter in 1983 to the first quarter in 1986. The sample only includes heads that were older than 16 years and younger than 70 years throughout the sample. The final sample has 8,170 observations.

Construction of the Aggregate Series. I created the consumption and income series from the National Income and Product Accounts largely following Blinder and Deaton (1985). The labor income series consists of labor and transfer income (the Citibase Series GW + GPOL + GPT) less social insurance contributions (GPSIN). To subtract the portion of taxes on labor income I created the ratio of wages, salaries and other labor income to income including interest, dividends and rents. Personal tax payments (GPTX) where multiplied by this ratio and the result subtracted from income. Proprietors' income (GPROP) was multiplied by the same ratio before adding it to the income series. Unlike Blinder and Deaton I did not add nontax payments to state and local governments to income and consumption because Citibase only reports this series starting from 1958. Income was adjusted in the second quarter of 1975 by subtracting the tax rebate and social security bonus. The amount of this adjustment is taken from Blinder (1981), table 2.

The real consumption series is constructed by adding the constant dollar expenditures on nondurables and services and subtracting expenditures on clothing and shoes because these have rather durable characteristics (GCN82 + GCS82 - GCNC82). The consumption deflator obtained by dividing the nominal consumption series by the real series is used to deflate income. Both income and consumption are first divided by the total population (GPOP) and then multiplied by the average number of household members. No quarterly series of average household size is available for the sample period. I used the figures pertaining to March from the Current Population Reports, Series P-20, No. 467 for the first quarter and interpolated the remaining quarters linearly. Since average household size is only changing very slowly, this approximation should be rather good.

Finally, to make the scale of the consumption series comparable to the income series it is multiplied by the ratio of total expenditures (GC82) to expenditures on nondurables and services. Quarterly NIPA series are reported at annual rates. I divided all series by four to obtain quarterly amounts.

Appendix C
Adjustment of the Quarterly Income Processes for Measurement Error

The aggregated quarterly observations for income I construct from the SIPP will generally draw information from one or two interviews. Given that an interview covers four months, the three months making up a quarter will be sequences of pairs (0,3), (1,2), (2,1), (3,0), where the first digit indicates the number of months coming from the first interview and the second the months from the next interview. After this sequence the pattern repeats. Due to the rotation group design, each pair will be represented about equally each quarter. The following table indicates how the process for observed quarterly income changes looks when the monthly observations pertain to each of the four possible patterns under the first measurement error model.

Table C1
Interview Structure and Income Processes for Measurement Error Model 1

Interview Overlap	Income Process
(0,3)	$\Delta y_{it} = \Delta y_{it}^{*} + 3\mu_{ij} - 3\mu_{ij-1}$
(1,2)	$\Delta y_{it} = \Delta y_{it}^{*} + 2\mu_{ij} - 2\mu_{ij-1}$
(2,1)	$\Delta y_{it} = \Delta y_{it}^{*} + \mu_{ij} - \mu_{ij-2}$
(3,0)	$\Delta y_{it} = \Delta y_{it}^{*} + 2\mu_{ij} - 2\mu_{ij-1}$

Starred income variables in table C1 refer to true income. The subscript t refers to quarters, j to interviews. Since the measurement error is uncorrelated across interviews and with true income, this yields the variances and autocovariances given in table C2. The calculations given in the rows labeled "average" are based on a value of 0.592 for σ_{μ}^{2}. All autocovariances beyond the second are zero.

Table C2 Interview Structure and Quarterly Variances and Autocovariances for Measurement Error in Model 1	
Interview Overlap	Covariance
(0,3)	$var(\Delta y_{it}) = var(\Delta y^*_{it}) + 18\sigma^2_\mu$
(1,2)	$var(\Delta y_{it}) = var(\Delta y^*_{it}) + 8\sigma^2_\mu$
(2,1)	$var(\Delta y_{it}) = var(\Delta y^*_{it}) + 2\sigma^2_\mu$
(3,0)	$var(\Delta y_{it}) = var(\Delta y^*_{it}) + 8\sigma^2_\mu$
average	$var(\Delta y_{it}) = var(\Delta y^*_{it}) + 9\sigma^2_\mu = var(\Delta y^*_{it}) + 5.33$
(0,3)	$cov(\Delta y_{it}, \Delta y_{it-1}) = var(\Delta y^*_{it}, \Delta y^*_{it-1}) - 6\sigma^2_\mu$
(1,2)	$cov(\Delta y_{it}, \Delta y_{it-1}) = var(\Delta y^*_{it}, \Delta y^*_{it-1}) - 6\sigma^2_\mu$
(2,1)	$cov(\Delta y_{it}, \Delta y_{it-1}) = var(\Delta y^*_{it}, \Delta y^*_{it-1}) + 2\sigma^2_\mu$
(3,0)	$cov(\Delta y_{it}, \Delta y_{it-1}) = var(\Delta y^*_{it}, \Delta y^*_{it-1}) - 2\sigma^2_\mu$
average	$cov(\Delta y_{it}, \Delta y_{it-1}) = var(\Delta y^*_{it}, \Delta y^*_{it-1}) - 3\sigma^2_\mu = var(\Delta y^*_{it}, \Delta y^*_{it-1}) - 1.78$
(0,3)	$cov(\Delta y_{it}, \Delta y_{it-2}) = var(\Delta y^*_{it}, \Delta y^*_{it-2}) - 3\sigma^2_\mu$
(1,2)	$cov(\Delta y_{it}, \Delta y_{it-2}) = var(\Delta y^*_{it}, \Delta y^*_{it-2})$
(2,1)	$cov(\Delta y_{it}, \Delta y_{it-2}) = var(\Delta y^*_{it}, \Delta y^*_{it-2}) - 3\sigma^2_\mu$
(3,0)	$cov(\Delta y_{it}, \Delta y_{it-2}) = var(\Delta y^*_{it}, \Delta y^*_{it-2}) - 4\sigma^2_\mu$
average	$cov(\Delta y_{it}, \Delta y_{it-2}) = var(\Delta y^*_{it}, \Delta y^*_{it-2}) - 2.5\sigma^2_\mu = var(\Delta y^*_{it}, \Delta y^*_{it-2}) - 1.48$

To calculate the adjustments necessary for the second measurement error model we first need to find the covariance structure of the time aggregated data under the assumption of an IMA(1,1) process for the monthly data. Let σ^2_ε be the monthly variance of the permanent component and σ^2_u the monthly variance of the transitory shock. This yields $var(\Delta y_{it}) = 19\sigma^2_\varepsilon + 6\sigma^2_u$ and $cov(\Delta y_{it}, \Delta y_{it-1}) = 4\sigma^2_\varepsilon - 3\sigma^2_u$ for the true quarterly process under the model while all other autocovariances are zero.

Observed quarterly income changes in the SIPP will follow the patterns given in table C3. Subscripts t refer to quarters while s refers to months and the dating is such that s is the last month in quarter t.

Table C3 Interview Structure and Income Processes for Measurement Error Model 2	
Interv. Overlap	Income Process
(0,3)	$\Delta y_{it} = 3(\varepsilon_{ijs+1} + \varepsilon_{ijs} + \varepsilon_{ijs-1} + \varepsilon_{ijs-2} + u_{ijs+1} - u_{ij-1s-3})$
(1,2)	$\Delta y_{it} = 2(\varepsilon_{ijs+2} + \varepsilon_{ijs+1} + \varepsilon_{ijs} + \varepsilon_{ijs-1} + u_{ijs+2} - u_{ij-1s-2})$
(2,1)	$\Delta y_{it} = \varepsilon_{ijs+3} + \varepsilon_{ijs+2} + \varepsilon_{ijs+1} + \varepsilon_{ijs} + \varepsilon_{ij-1s-1} + \varepsilon_{ij-1s-2} + \varepsilon_{ij-1s-3} + \varepsilon_{ij-1s-4} + u_{ijs+3} - u_{ij-2s-5}$
(3,0)	$\Delta y_{it} = 2(\varepsilon_{ijs} + \varepsilon_{ijs-1} + \varepsilon_{ijs-2} + \varepsilon_{ijs-3} + u_{ijs} - u_{ij-1s-4})$

This implies that the measured quarterly income process has the covariance structure given in table C4.

Table C4 Interview Structure and Quarterly Variances and Autocovariances of Measured Income for Measurement Error Model 2	
Interv. Overlap	Covariance
(0,3)	$var(\Delta y_{it}) = 36\sigma_{\varepsilon}^2 + 18\sigma_u^2$
(1,2)	$var(\Delta y_{it}) = 16\sigma_{\varepsilon}^2 + 8\sigma_u^2$
(2,1)	$var(\Delta y_{it}) = 8\sigma_{\varepsilon}^2 + 2\sigma_u^2$
(3,0)	$var(\Delta y_{it}) = 16\sigma_{\varepsilon}^2 + 8\sigma_u^2$
average	$var(\Delta y_{it}) = 19\sigma_{\varepsilon}^2 + 9\sigma_u^2 = var(\Delta y_{it}^{*}) + 3\sigma_u^2 = var(\Delta y_{it}^{*}) + 1.50$
(0,3)	$cov(\Delta y_{it}, \Delta y_{it-1}) = -6\sigma_u^2$
(1,2)	$cov(\Delta y_{it}, \Delta y_{it-1}) = -6\sigma_u^2$
(2,1)	$cov(\Delta y_{it}, \Delta y_{it-1}) = 8\sigma_{\varepsilon}^2 + 2\sigma_u^2$
(3,0)	$cov(\Delta y_{it}, \Delta y_{it-1}) = 8\sigma_{\varepsilon}^2 + 2\sigma_u^2$
average	$cov(\Delta y_{it}, \Delta y_{it-1}) = 4\sigma_{\varepsilon}^2 - 2\sigma_u^2 = cov(\Delta y_{it}^{*}, \Delta y_{it-1}^{*}) + \sigma_u^2 = cov(\Delta y_{it}^{*}, \Delta y_{it-1}^{*}) + 0.50$
(0,3)	$cov(\Delta y_{it}, \Delta y_{it-2}) = -6\sigma_u^2$
(1,2)	$cov(\Delta y_{it}, \Delta y_{it-2}) = 0$
(2,1)	$cov(\Delta y_{it}, \Delta y_{it-2}) = -3\sigma_u^2$
(3,0)	$cov(\Delta y_{it}, \Delta y_{it-2}) = -4\sigma_u^2$
average	$cov(\Delta y_{it}, \Delta y_{it-2}) = -3.25\sigma_u^2 = cov(\Delta y_{it}^{*}, \Delta y_{it-2}^{*}) - 3.25\sigma_u^2 = cov(\Delta y_{it}^{*}, \Delta y_{it-2}^{*}) = -1.63$

Appendix D

Calculations of Utility Loss

In this appendix I discuss how to calculate the utility loss the household suffers by ignoring aggregate information in consumption decisions. The basic setup is taken from the appendix in Cochrane (1989, pp. 334-335). The second part gives the matrix representations of the full information model and the no information model used in the utility calculations.

Utility for the quadratic model can be written as

$$U(X_t) = E_t \sum_{j=0}^{\infty} \beta^j X_{t+j}' R X_{t+j} \tag{D1}$$

where $\beta = 1/(1+r)$ and X_t represents the state vector of the system which evolves according to

$$X_t = AX_{t-1} + \Gamma\xi_t \tag{D2}$$

$$E_t(\xi_{t+1}) = 0$$

$$E_t(\xi_t \xi_t') = \Sigma$$

Equation (D1) can be rewritten as

$$U(X_t) = X_t' P X_t + \frac{1+r}{r}\text{Trace}(P\Gamma\Sigma\Gamma') \tag{D3}$$

where

$$P = R + \beta A' P A \tag{D4}$$

P will be a symmetric matrix; therefore (D4) cannot be solved directly P. Cochrane shows, however, that

$$M\text{vec}(P) = (I - \beta M(A' \otimes A')N)^{-1} M\text{vec}(R) \tag{D5}$$

where M is a transformation matrix that deletes the redundant rows of a stacked symmetric matrix and N does the opposite operation, i.e.

$$\text{vech}(P) = M\text{vec}(P)$$

$$N\text{vech}(P) = \text{vec}(P)$$

Cochrane uses (D3) and (D5) to solve analytically for $U(X_t)$. Instead, once the model is expressed in the form (D1) and (D2), these equations can easily be used to calculate utility numerically. Due to the complexity of the models I

took this latter route. Since the quantity of interest is the difference between lifetime utility for two alternative models it is rather small compared to total utility. Computational inaccuracies can therefore play a large role in these numerical calculations. The results should therefore be taken as indicative of magnitudes rather than as precise amounts.

The full information model. Instead of comparing the no information model to Goodfriend's model with lagged information I chose to use a model with full contemporaneous information on aggregate variables as the benchmark. This model will yield higher utility than Goodfriend's. The utility comparisons I present will therefore be upper bounds for the choice relevant to the consumer.

Since all the variables refer to a single household and the distinction between aggregate and individual variables is not important here I suppress i subscripts for notational convenience. Income in the full information model is given by the first line in (37) in the text.

$$\Delta y_t = (1+\phi_1 L+\phi_2 L^2)\varepsilon_t + (1-\alpha_1 L-\alpha_2 L^2)u_t \tag{D6}$$

Optimal consumption is given by

$$\begin{aligned} c_t &= \frac{r}{1+r}\left[W_t + \sum_{i=0}^{\infty}\frac{E_t\, y_{t+i}}{(1+r)^i}\right] \\ &= \frac{r}{1+r}W_t + y_t + \left[\frac{\phi_1}{1+r}+\frac{\phi_2}{(1+r)^2}\right]\varepsilon_t + \frac{\phi_2}{1+r}\varepsilon_{t-1} - \left[\frac{\alpha_1}{1+r}+\frac{\alpha_2}{(1+r)^2}\right]u_t - \frac{\alpha_2}{1+r}u_{t-1} \end{aligned} \tag{D7}$$

and assets follow

$$\begin{aligned} W_t &= (1+r)\left[W_{t-1} + y_t - c_t\right] \\ &= W_{t-1} - \left[\phi_1+\frac{\phi_2}{1+r}\right]\varepsilon_{t-1} - \phi_2\varepsilon_{t-2} + \left[\alpha_1+\frac{\alpha_2}{1+r}\right]u_{t-1} + \alpha_2 u_{t-2} \end{aligned} \tag{D8}$$

Define the state vector as

$$X_t = [1 \;\; W_t \;\; y_t \;\; \varepsilon_t \;\; \varepsilon_{t-1} \;\; u_t \;\; u_{t-1}]' \tag{D9}$$

Using (D7) and (D9) we can write

$$c_t - \bar{c} = \left[-\bar{c} \quad \frac{r}{1+r} \quad 1 \quad \frac{\phi_1}{1+r}+\frac{\phi_2}{(1+r)^2} \quad \frac{\phi_2}{1+r} \quad -\left[\frac{\alpha_1}{1+r}+\frac{\alpha_2}{(1+r)^2}\right] \quad -\frac{\alpha_2}{1+r}\right]X_t \equiv F'X_t \tag{D10}$$

Then R in (D1) is given by

$$R = -\frac{1}{2}FF' \tag{D11}$$

The transition equation for the system in (D2) becomes

$$\begin{bmatrix} 1 \\ W_t \\ y_t \\ \varepsilon_t \\ \varepsilon_{t-1} \\ u_t \\ u_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\left(\phi_1 + \frac{\phi_2}{1+r}\right) & -\phi_2 & \alpha_1 + \frac{\alpha_2}{1+r} & \alpha_2 \\ 0 & 0 & 1 & \phi_1 & \phi_2 & -\alpha_1 & -\alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ W_{t-1} \\ y_{t-1} \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \\ u_{t-1} \\ u_{t-2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix} \tag{D12}$$

The no information model. The income process to the household in the no information model looks like

$$\Delta y_{it} = (1 - \theta_1 L - \theta_2 L^2)\eta_t \tag{D13}$$

Consumption is given by

$$c_t = \frac{r}{1+r} W_t + y_t - \left[\frac{\theta_1}{1+r} + \frac{\theta_2}{(1+r)^2}\right]\eta_t - \frac{\theta_2}{1+r}\eta_{t-1} \tag{D14}$$

and assets follow

$$W_t = W_{t-1} + \left[\theta_1 + \frac{\theta_2}{1+r}\right]\eta_{t-1} + \theta_2 \eta_{t-2} \tag{D15}$$

Define the state vector as

$$X_t = [1 \;\; W_t \;\; y_t \;\; \eta_t \;\; \eta_{t-1}]' \tag{D16}$$

Using (D14) and (D16)

$$c_t - \bar{c} = \left[-\bar{c} \;\; \frac{r}{1+r} \;\; 1 \;\; -\left[\frac{\theta_1}{1+r} + \frac{\theta_2}{(1+r)^2}\right] \;\; -\frac{\theta_2}{1+r}\right] X_t \equiv F' X_t \tag{D17}$$

The transition equation becomes

$$\begin{bmatrix} 1 \\ W_t \\ y_t \\ \eta_t \\ \eta_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \theta_1 + \frac{\theta_2}{1+r} & \theta_2 \\ 0 & 0 & 1 & -\theta_1 & -\theta_2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ W_{t-1} \\ y_{t-1} \\ \eta_{t-1} \\ \eta_{t-2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \eta_t \tag{D18}$$

Once both models have been solved for the level of utility attained the utility difference is converted to quarterly rates by multiplying by $r/(1+r)$. To convert the utility loss to dollar terms divide the utility loss by the expected value of marginal instantaneous utility

$$\$\text{ loss / quarter} = \frac{r}{1+r}\,\frac{\Delta U}{Eu'(c_t)} = \frac{r}{1+r}\,\frac{\Delta U}{(\bar{c}-\bar{y})} = \frac{r}{1+r}\,\frac{\gamma \Delta U}{\bar{y}} \tag{D19}$$

where γ is the coefficient of relative risk aversion. The calculations in the paper are for a coefficient of relative risk aversion of two and a mean income level of \$6,902.

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